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Screening in three-dimensional QED with arbitrary fermion mass

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Abstract. We compute the quark–antiquark potential in three-dimensional massive quantum electrodynamics for an arbitrary fermion mass. The result indicates that screening prevails for any quark masses, contrary to the classical expectations, generalizing our previous result obtained for large masses. We also test the validity of several approximation schemes using a detailed numerical analysis. The classical result is still reproduced for a small separation of the quarks.

1 Introduction

A proper study of the problem of screening and confinement is of considerable importance in our understanding of gauge theories. To avoid the complexities of four dimensions these studies are usually confined to lower dimensions. In this framework, a deep physical interpretation has been achieved. Indeed, in two-dimensional QED [1], one obtains screening in the massless case, but confinement in the massive quark case, realizing the expected picture.

For QCD in two dimensions Gross et al. [2] were the first to discuss the subject. If dynamical fermions and test charges are in different representations, they find screening or confinement in some particular cases depending on whether the fermion is massless or massive. A similar conclusion in an identical setting has been arrived at for the massless case in [3]. If, on the other hand, all fermions are in the fundamental representation, then screening prevails independently of the quark mass [4].

General inquiries in two-dimensional gauge theories have been performed recently by several authors [5], concerning the θ -vacuum structure, screening, confinement and chiral condensates. In three dimensions related questions were studied in [6].

In three-dimensional space-time, for an abelian gauge group, the question of screening versus confinement has recently been analysed for large fermion masses [7] in which case the fermionic determinant can be computed as a series in the inverse mass. The conclusion was that, contrary to classical expectations, the theory is in the screening phase. Although this is expected from the fact that a Chern–Simons term develops and there is a topological mass generation, it is a further indication that the dynamics of gauge fields and the deep problem of screening versus confinement is far from being settled by a simple inspection of the classical behaviour of the theory. In the case of three-dimensional QED, the outcome reveals that the vacuum polarization is once more capable of developing configurations that screen the external quarks, presumably modifying the dynamics of quark–antiquark bound states.

Here we extend the analysis of our previous work [7] in order to include all values of the fermion mass parameter. An explicit expression for the quark–antiquark potential is obtained following the usual ideas of bosonisation [1,8] but an analytic form cannot be obtained, and we resort to the use of numerical analysis. The results show that the screening phase obtained in the large mass limit [7] persists for any value of the mass parameter (including vanishing mass). It may be mentioned that although the occurrence of a Chern–Simons term in the large mass limit suggests screening, the absence of such a term for arbitrary mass clearly implies that the phenomenon of screening is independent of its presence or absence. Next, the validity of certain approximation schemes [9] is tested. Using these approximations a simple form of the quark–antiquark potential can be given, which is compared with the exact form. We still obtain screening; moreover, the behaviour of the functions is very accurately described by the proposed approximations.

This paper is divided as follows. In Sect. 2 we present the computation of the potential and the numerical results. We draw the potential for different values of the mass parameter, showing that its form is essentially the same in the whole range of real values for the mass term, leading to the screening phase. Furthermore, we test the approximation forms of all the functions necessary for computing the potential. Section 3 is reserved for conclusions and discussions. Especially important in the last section is the comparison between our result and a similar computation using Wilson loops, leading to exactly the

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same potential in the case where both procedures are feasible [10].

2 Computation of the quark–antiquark potential

The partition function of three-dimensional massive QED in the covariant gauge, in the presence of an external source J^μ, is given by

$$Z = \int \mathcal{D}[\psi, \bar{\psi}, A_\mu]\delta(\partial_\mu A^\mu) \times \exp\left\{ \mathrm{i}\int \mathrm{d}^3x \left[\bar{\psi}(\mathrm{i}\not{\partial} - m - e\not{A})\psi - \frac{1}{4}F^2_{\mu\nu} + A_\mu J^\mu\right]\right\}, \tag{1}$$

where $F_{\mu\nu}$ is the field tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The bosonised version of the above defined action in the weak coupling approximation is given by the expression [11,12]

$$Z = \int \mathcal{D}A_\mu \delta(\partial_\mu A^\mu) \tag{2}$$
$$\times \exp\left\{\mathrm{i}\int \mathrm{d}^3x\left\{\frac{1}{2}A_\mu \Pi^{\mu\nu}A_\nu - \frac{1}{4}F^2_{\mu\nu} + A_\mu J^\mu + \cdots\right\}\right\},$$

where the dots stand for non-quadratic terms in the gauge field A_μ. This result will describe the partition function of the Maxwell–Chern–Simons [13] theory in the covariant gauge in the infinite mass limit [7], as we see from the explicit expression for the self-energy of the gauge field, $\Pi_{\mu\nu}$, given by the expression

$$\Pi_{\mu\nu} = H(p)\mathrm{i}\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2} + \left[G(p) + p^2\right]\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right), \tag{3}$$

where the functions G and H are given by

$$H(p) = -\frac{e^2p^2}{4\pi}\int_0^1 \mathrm{d}t \frac{m}{\{m^2 - t(1-t)p^2\}^{1/2}}, \tag{4}$$

$$G(p) = -p^2 - \frac{\mathrm{e}^2p^2}{2\pi}\int_0^1 \mathrm{d}t\frac{t(1-t)}{\{m^2 - t(1-t)p^2\}^{1/2}}, \tag{5}$$

and $p = (-p^2)^{1/2}$. We compute the potential as being the difference between the Hamiltonian with and without a pair of static external charges separated by a distance L,

$$\begin{aligned}2V(L) = H_q - H_0 &= -(L_q - L_0)\\ &= -q\int \mathrm{d}^2x A_\mu \delta^{\mu 0}\{\delta(x^1 + L/2)\delta(x^2)\\ &\quad - \delta(x^1 - L/2)\delta(x^2)\}\\ &= -q\{A_0(x^1 = -L/2, x^2 = 0)\\ &\quad - A_0(x^1 = L/2, x^2 = 0)\},\end{aligned} \tag{6}$$

where we have integrated over the two space components in order to find the potential, and we considered the source as corresponding to two fixed charges of magnitude q located at the points defined by the respective delta functions. Note that $L_q(L_0)$ denote the Lagrangians in the presence (absence) of the charges.

We now consider the equations of motion associated with the action defined by means of (2). The field equation in the covariant gauge reads

$$-H(\partial = \sqrt{-\partial^2})\epsilon_{\mu\nu\rho}\frac{\partial^\nu}{\partial^2}A^\rho + G(\partial)\left(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)A^\nu + J_\mu = 0. \tag{7}$$

Defining the curl of A_μ as

$$\mathcal{A}_\mu = -\epsilon_{\mu\nu\beta}\partial^\nu A^\beta, \tag{8}$$

the equation of motion can be expressed as

$$\{\Box_{\mathrm{nonloc}} + m^2_{\mathrm{nonloc}}\}\mathcal{A}_\mu = -\epsilon_{\mu\nu\beta}\partial^\beta J^\nu - f_{\mathrm{nonloc}}J_\mu, \tag{9}$$

where

$$\begin{aligned}\Box_{\mathrm{nonloc}} &= G(\partial),\\ m^2_{\mathrm{nonloc}} &= \frac{H^2(\partial)}{\partial^2 G(\partial)},\\ f_{\mathrm{nonloc}} &= \frac{H(\partial)}{G(\partial)}.\end{aligned} \tag{10}$$

In the absence of sources, and in the large m limit [7], it reproduces the familiar massive mode of Maxwell–Chern–Simons theory [13]. From (6) it is seen that an expression for A_0 is required to calculate the potential. This is given in terms of the curl (8) by

$$\mathcal{A}_2 = -\partial_1 A_0. \tag{11}$$

The time independent solution for $\mathcal{A}_2$ corresponding to the sources describing static quarks can be obtained from (9). Using this result with (11) finally yields, after integrating over the angular variables,

$$A_0(t, L) = A_0(0, L) = -\frac{q^2}{\pi}\int_0^\infty \frac{kJ_0(kL)}{\tilde{G}(k) + \frac{\tilde{H}^2(k)}{k^2\tilde{G}(k)}}\mathrm{d}k. \tag{12}$$

The integration over the angular variables in the Fourier transformation led to the Bessel function $J_0(kL)$; in the case where the denominator is given by the familiar result (i.e. the Feynman propagator, which also appears in the large mass limit, see [7]) the result of the integration is just the modified Bessel function [14].

The potential is now found from (6), (9) and (12), reading

$$V(L) = -\frac{q^2}{2\pi}\int_0^\infty \frac{kJ_0(kL)}{\tilde{G}(k) + \frac{\tilde{H}^2(k)}{k^2\tilde{G}(k)}}\mathrm{d}k, \tag{13}$$

where the functions $G(k)$ and $H(k)$ are given by the expressions

$$G(k) = k^2 \tag{14}$$

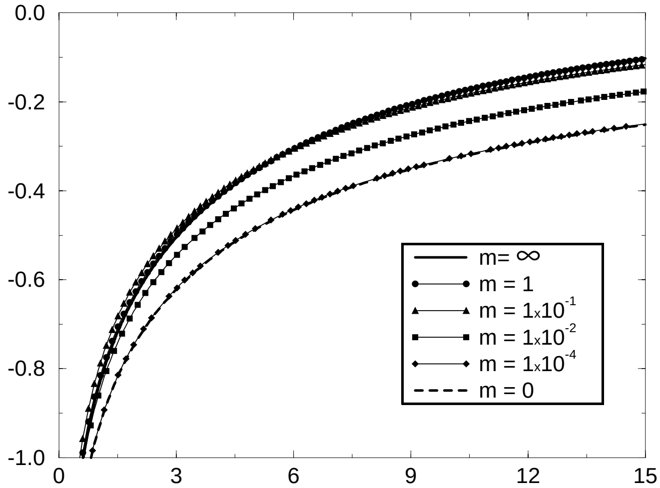

Fig. 1. Potential as a function of the distance for various values of the mass parameter. The value $m = 1$ practically coincides with the asymptotic value $m = \infty$

$$+\frac{e^2 m}{4\pi}\left[1-\left(\frac{2m}{k}-\frac{k}{2m}\right)\arctan\left(\frac{k}{2m}\right)\right],$$
$$H(k) = \frac{e^2 mk}{2\pi}\arctan\left(\frac{k}{2m}\right). \tag{15}$$

The above equation takes a particularly simple form in the infinite mass limit,

$$V(L) = -\frac{1}{2\pi}\frac{q^2}{1+\frac{e^2}{6\pi m}}K_0\left(\frac{e^2}{4\pi}L\right) \equiv -\frac{q^2_{\mathrm{ren}}}{2\pi}K_0\left(\frac{e^2}{4\pi}L\right). \tag{16}$$

This reproduces our earlier results in [7]. The asymptotic form of the Bessel function signals screening.

For arbitrary mass however, a simple closed form expression cannot be obtained. We therefore have to use numerical methods. They are presented as follows. We first plot the function $V(L)$ given in (13) as a function of L for various values of the mass parameter m. The result is plotted in Fig. 1. It is immediately obvious that the screening effect is qualitatively independent of the mass, and the quantitative dependence extremely small. Indeed, the graphs are almost bound inside a rather narrow band defined by the results obtained for $m = 0$ and $m = \infty$.

In general, due to the appearance of non-algebraic functions the expressions appearing in (14) and (15) are rather clumsy. In [9] simple expressions have been derived which, according to the authors, give a good approximation to these functions in the whole range of values of the parameter. The approximations are

$$G(k) \approx k^2\left\{1+\frac{1}{16}\left[k^2+\left(\frac{3\pi m}{4}\right)^2\right]^{-1/2}\right\}, \tag{17}$$
$$H(k) \approx \frac{e^2 mk^2}{4}\left(k^2+\pi^2 m^2\right)^{-1/2}. \tag{18}$$

We tested this assumption for the computation of the potential, comparing the asymptotic result with the one obtained with the approximations for $m = 1000$. We repeated the procedure for $m = 0.1$, which shows similar findings. The result is shown in Fig. 2, indicating that the approximation agrees remarkably well with the exact results. We also checked the approximations directly in Fig. 4. Comparison of the expressions for G and H using the approximations and the exact result shows that there is little discrepancy.

The approach to the asymptotes in the computation of the potential has also been analysed. We have verified that it is very quick. Indeed, for reasonably low values of the mass the potential already shows the asymptotic value. We illustrated this behaviour in the case $m = 10$ in Fig. 3, which was done with the exact expressions.

3 Discussion

Here we worked out an approach for obtaining the semi-classical interquark potential for arbitrary values of the fermion mass parameter, generalising our previous work [7], where only the infinite mass approximation was analysed. In that case, the expression for the effective action is local, and the interquark potential could be computed in closed form, showing explicitly that the model lives in a screening phase. We attempted here to go beyond the large mass limit. However, the expressions thus obtained are non-local, and we had to resort to numerical simulations. The results are nevertheless rewarding, especially in view of the extremely mild dependence upon the fermion mass, i.e. almost every physical quantity related to the screening potential is almost independent of the mass, for $0 \le m \le 1$ (we suppose $e = 1$) and reaches the asymptotic value already for m of order unit.

The screening obtained for all values of the mass parameter supports the observations obtained in two-dimensional QCD [4], where the screening phase also prevails almost universally (see also [2]). This opens up the discussion for a large number of interesting possibilities. In particular, it is interesting to stress that the same mechanism may work for the non-abelian case in three dimensions, since in the large mass limit the effective action turns out to be the non-abelian generalization of the Maxwell–Chern–Simons theory. In an axial gauge, and in the weak coupling limit, the Maxwell–Chern–Simons action coincides with the abelian counterpart, and the same conclusions are expected. If one dares to speculate that the mass dependence is as mild as we have obtained in the above discussion, then all conclusions can be carried over to the non-abelian case as well, a tantalising result! This would imply an almost universal screening behaviour for low dimensional systems. We hope to come back to these interesting questions in a future work.

The derivation of the potential from the process of computing the determinant of the Dirac operator is a standard one [17], being based on perturbation theory. The fact that the coupling constant has a positive mass dimension makes this procedure possible. Nonetheless, we are aware of the fact that in the large mass case the perturbative expansion leads to the best results, as mentioned

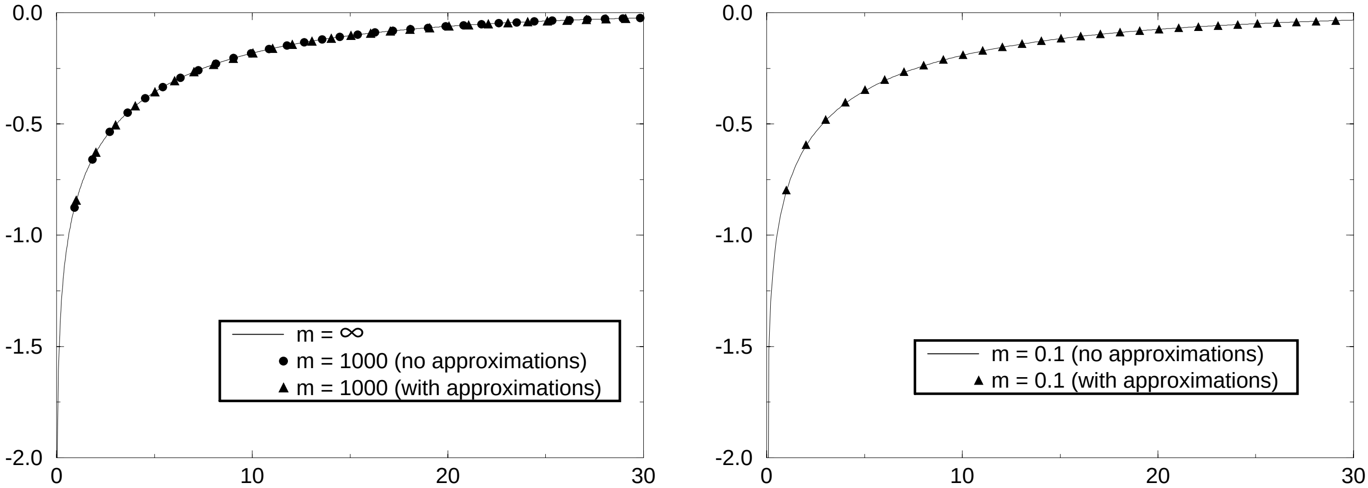

Fig. 2. (Left) Potential function for infinite mass, and for $m = 1000$ using (17) and (18). (Right) Potential function for $m = 0.1$ using the exact results (14) and (15) and approximate forms (17) and (18)

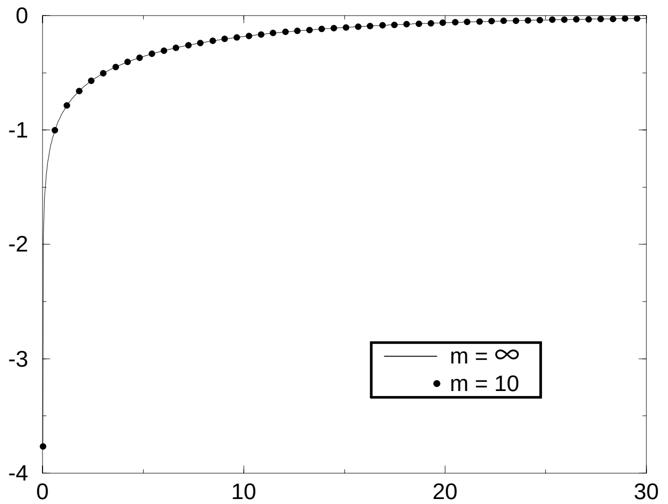

Fig. 3. Comparison between the potential for $m = 10$ and the asymptotic result

several times in the conclusion. What we pointed out is that, at lowest order of perturbation theory the mass term is irrelevant, as shown in Fig. 1. This is by all means important, since lowest order perturbation is still valid to finite mass (we are *not* talking about the zero mass limit!). We certainly have to go beyond the quadratic approximation as a next step. We nevertheless consider the fact that the mass parameter does not play a fundamental role in the present calculation, giving a strong indication that screening may prevail in much more general circumstances than usually believed. Our computation for zero mass does not constitute any proof, but is just performed to show a trend. We do not and cannot believe that the zero mass limit provides good results, and is only displayed as a bias towards stabilisation of the screening process. This is in fact confirmed by the independent computation in [18].

We also tested the approximative formulae (17) and (18), usually taken as a good approximation within 10% accuracy. The difference between the potential calculated using these approximations and the exact results is less than 10^{-2}, too small to be seen in Fig. 2.

Let us now digress a bit on the infinite and zero mass limits of the model. In the latter case we have to go beyond the quadratic approximation, since higher corrections have to be computed in a gauge theory where further powers of the external momenta show up in the computation of the diagrams. One loop fermionic diagrams in gauge theories result in powers of momentum and functions of momentum over mass, in general $f\left(p^2/m^2\right)$. While the limit $p \to 0$ and $m \to \infty$ is unambiguous, the double limit $p \to 0$ and $m \to 0$ is not well defined, and depends on the order in which they are taken. Therefore, a strong infrared dependence on the mass may invalidate the procedure. Notice that the discussion of screening presupposes a large distance (namely a small momentum) limit, which may not commute with the zero mass limit. The infinite mass limit obtained in the quadratic approximation is however expected to survive even in the non-quadratic regime. We hope to come back to these points in a future publication.

We finally comment on two important points which make contact with the existing literature on the problem of screening and confinement, namely the existence of monopole solutions [15] and the Wilson loop formulation of the confinement problem [16].

In the first case, we know that for QED_3 without fermions there are monopole solutions, such as described by e.g. the vector potential $A_r = (g/r)\varphi$, $A_0 = A_\varphi = 0$. However, we see that the quantum effect described by the mechanism described following (6) acts here in the same way so as to make the above result disappear from the space of solutions. Indeed we can introduce an interaction similar to (6) in terms of A_r.

The fact that the mechanism in action here is of a quantum nature, independent of the details of the Chern–Simons term can also be confirmed from the very recent paper [18], where the authors showed that even *bosonic*

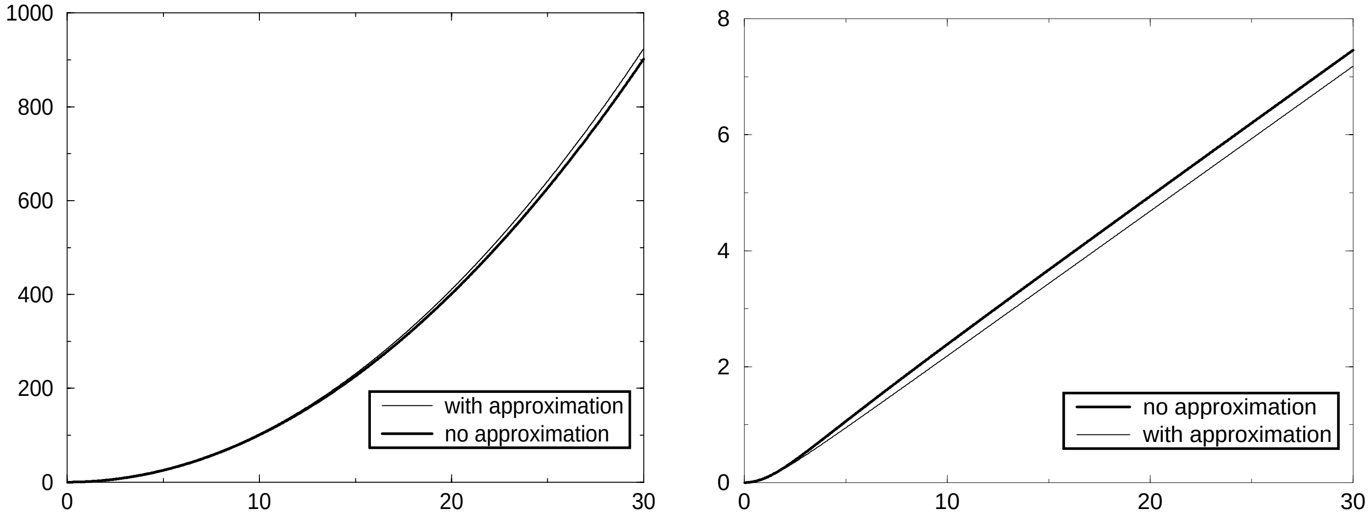

Fig. 4. Function G (left) and H (right) with $m = 1$, using the exact definitions (14) and (15), and the approximations (17) and (18)

matter is screened rather than confined in three-dimensional QED.

A further check of our result, rather detailed and fine from the dynamical point of view has been performed by K.D. Rothe [10]. The Wilson loop approach is very difficult to be performed in the arbitrary mass case, but for a large mass this can be done and the potential can be obtained. In our case the large mass limit is such that the potential boils down to an expression in terms of a Bessel function (16), while the effective Lagrangian is just the Maxwell–Chern–Simons Lagrangian, as we can easily see from e.g. (3). The Wilson loop expectation is given by [10]

$$\langle W[\mathcal{C}]\rangle = \int \mathcal{D}A_\mu \mathrm{e}^{S+\mathrm{i}e\int I_\mu A^\mu \mathrm{d}^3 z}, \tag{19}$$

where I_μ defines the charge current at the border of the Wilson loop. One finds from the quadratic part of the action the result

$$\langle W\{\mathcal{C}\}\rangle = \mathrm{e}^{\frac{e^2}{2}\int I^\mu K^{-1}_{\mu\nu} I^\nu}. \tag{20}$$

Choose the Wilson loop $\mathcal{C}$ to be a square $-T/2 \le x^0 \le T/2$, $-L/2 \le x^1 \le L/2$, $x^2 = 0$. The inverse propagator $K^{-1}_{\mu\nu}$ is given in terms of the function $\Delta(x, M)$ (where M is the Chern–Simons mass as we have computed it) obeying the Klein–Gordon equation. Its computation leads to the result (16) for the potential, fully confirming, via the Wilson loop procedure, the correctness of the result. Further $1/m$ corrections can be computed accordingly.

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